Multi-particles continued

1.)

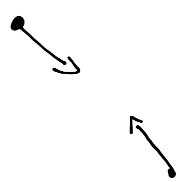
$\sum \vec{F}^{ext} = \dot{\vec{L}}$ $\quad \vec{L} = \sum m_i \underline{\vec{V}_i} \rightarrow \vec{V}_{i/F}$

$\sum \vec{M}^{ext}_{/c} = \sum m_i \vec{r}_{i/c} \times \vec{a}_{i/F}$

2.) Equivalent to 1.)

Assume: $\sum \vec{F}^{int} = 0$ $\quad \sum \vec{M}_{/c} = 0$ $\quad \vec{F}^{tot} = m\vec{a}$ for each particle

3.) Classical Approach



pairwise
central
equal · opposite
forces

a) Microscopic Assumption
(none of our business)

b.) wrong physics

c.) Bad macroscopic predictions
e.g. $\nu = 1/4$ (poisson's ratio)

Angular Momentum

$$\boxed{\sum \vec{M}_{/c} = \sum \vec{r}_{i/c} \times (m\vec{a}_{i/F})} \longrightarrow \text{A.M.B.}$$

$\sum \vec{M}_{/c} = \frac{d}{dt} \vec{H}_{/c}$, if we define $\vec{H}_{/c}$ appropriately

What are good definitions of $\vec{H}_{/c}$?

c is a fixed point (fixed in any Newtonian frame)

$\vec{V} = \vec{V}_{/c} = \vec{V}_{/F}$ $\vec{H}_{/c} = \sum \vec{r}_{i/c} \times m \underline{\vec{V}_{i/c}} \to \vec{V}_{i/F}$

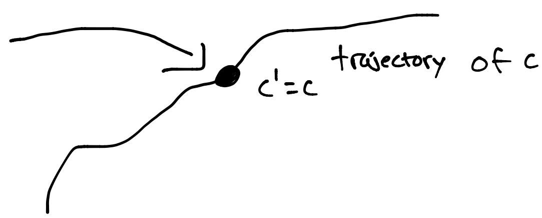
③ $C = G = COM = \sum r_i m_i / m_{total}$

$\vec{H}_{/G} = \sum \vec{r}_{i/G} \times m_i \vec{V}_{i/G}$

$\sum \vec{M}_{/G} = \dot{\vec{H}}_{/G}$

③ $\vec{H}_{/c} = \vec{r}_{i/c'} \times m_i \vec{V}_{i/F}$ c' is a fixed point instantaneously coinciding with c

at t of intersection c=c'



<u>Summary</u>

define $\vec{H}_{/c}$ If $\frac{d}{dt}\vec{H}_{/c} = \sum \vec{r}_{i/c} \times m\vec{a}_{i/F}$ good

else bad

<u>Energy</u>

$\vec{F}_i = m_i \vec{a}_i$ true for every particle

$\vec{V}_i \cdot \vec{F}_i = m_i \vec{V}_i \cdot \vec{a}_i$

$\sum \vec{V}_i \cdot \vec{F}_i = \sum m_i \vec{v}_i \cdot \vec{a}_i$ $\vec{F}_i = \vec{F}_i^{int} + \vec{F}^{ext}$

$$\boxed{P^{int}_{power} + P^{ext}_{power} = \dot{E}_K} \qquad E_K = \sum \frac{1}{2} m_i v_i^2$$

Separate internal and external forces into conservative and non-conservative forces

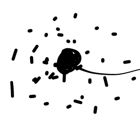
- associate potential energy with them. We get:

$$\underbrace{P^{ext}_{nc} + P^{int}_{nc}}_{\text{nonconservative}} = \dot{E}_K + \underline{\dot{E}_P} \rightarrow \sum E_P^{int} + E_P^{ext}$$

Often people assume:

$$\boxed{\begin{aligned} P^{int}_{n.c} &= \underbrace{\dot{D}}_{\text{dissipation}} \geq 0 \\ P^{ext}_{n.c} - \dot{D} &= \dot{E}_{tot} \end{aligned}}$$

<u>Center of Mass</u>

 G = c.o.m.

average position of mass

$$m_{tot}\vec{r}_G = \sum \vec{r}_i m_i$$

Use center of mass to simplify expressions: $\vec{L}, \vec{H}/_c, E_K$

$$\dot{\vec{L}} = \sum m_i \vec{a}_i = \frac{d}{dt}(m_{tot}\vec{V}_G)$$
$$= \frac{d}{dt}\sum m_i \vec{V}_i$$

$$\boxed{\vec{L} = \frac{d}{dt}\vec{r}_G\, m_{total}}$$

$\vec{H}_{/c} = \sum \vec{r}_{i/c} \times m_i \vec{v}_{i/c}$ c = a fixed point

– simplify with center of mass

Back to $\vec{L}$, $\vec{L} = m_i \underline{\vec{v}_i} \rightarrow \vec{v}_G + \vec{v}_{i/G}$

$$\vec{L} = \sum m_i \vec{v}_G + \sum m_i \vec{v}_{i/G}$$

$\dot{r}_i \; \frac{d}{dt}$

$$= \sum m_i \vec{v}_G + \sum m_i \vec{v}_i - \sum m_i \vec{v}_G$$

$$= \sum m_i \vec{v}_G + m_{tot} \vec{v}_G - m_{tot} \vec{v}_G$$

$$\vec{L} = \sum m_i \vec{v}_G = m_{tot} \vec{v}_G$$

Back to angular:

$\vec{r}_{i/c} = \vec{r}_{G/c} + \vec{r}_{i/G}$

$\vec{v}_{i/c} = \vec{v}_{G/c} + \vec{v}_{i/G}$

$$\vec{H}_{/c} = \sum (\vec{r}_{G/c} + \vec{r}_{i/G}) \times (\vec{v}_{G/c} + \vec{v}_{i/G})\, m_i$$

2 of 4 terms are zero

see $\vec{L}$ calculation for reasoning

$$\vec{H}_{/c} = \vec{r}_{G/c} \times m_{tot} \vec{v}_G + \sum \vec{r}_{i/G} \times \vec{v}_{i/G}\, m_i$$

$$= \vec{H}_{G/c} + \vec{H}_{/G}$$